Smoothing Speech Trajectories by Regularization

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Abstract

The articulators of human speech might only be able to move slowly, which results in the gradual and continuous change of acoustic speech properties. Nevertheless, the so-called speech continuity is rarely explored to discriminate different phones. To exploit this, this paper investigates a multiple-frame MFCC representation (that is expected to retain sufficient time-continuity information) in combination with a supervised dimensionality reduction method, whose target is to find low-dimensional representations that optimally separates different phone classes. The speech continuity information is integrated into this framework by using the regularization terms that penalize discontinuities. Experimental results on TIMIT phonetic classification show that the use of regularizers can help to improve the separability of phone classes.

Index Terms: Dimensionality Reduction; Contextual Representation; TIMIT Phone Classification; Regularization; Laplacian Smoothing;

1. Introduction

Speech is generated by (semi-)continuous movements of a small number of articulators, each of which are characterized by a small number of degrees of freedom. This suggests that speech signals can be adequately represented by a small number of parameters that vary in a smooth manner as a function of time. Plosives form the single most conspicuous exception to this pattern, because these sounds are characterized by sudden changes in the articulatory system. Still, capturing the articulatory dynamics in the speech signals holds the promise of improving acoustic modeling in automatic speech recognition (ASR) [1–6]. Specifically, capturing trajectories might eliminate the trajectory folding phenomenon in conventional ASR systems [4, 7]. However, despite promising advances in recovering articulatory gestures from the acoustic speech signals (e.g., [8, 9]), purely articulatory-based automatic speech recognition is not yet feasible (and it may never be). Therefore, we need to recur to techniques that allow approximating articulatory dynamics using acoustic representations.

Probably the simplest way for creating representations that represent dynamic articulatory gestures is by concatenating a number of consecutive 10 ms frames. However, such a block of frames captures the underlying gestures only implicitly. If we want to represent syllable-sized pieces of a speech signal, we need approximately 25 frames, for an average syllable duration of 250 ms. If each frame consists of 13 MFCCs (or a similar number of Mel-frequency spectral coefficients) each block comprises over 300 numbers. This is surely a heavily redundant representation, so that there is an obvious need for dimensionality reduction. Ideally, the dimensionality reduction method should help to highlight the dynamic articulatory gestures that have produced the speech signal.

The speech signals that can be generated by continuous movements of articulators that have a only few degrees of freedom will probably be on some low-dimensional manifold in the very high-dimensional acoustic space. Manifold learning [10] has proven to be effective for analyzing trajectory-based signal representations in video processing [11], dynamic texture analysis [12], and speech processing [13]. In [14, 15] it has been shown that manifolds of speech trajectories are effective in separating the overlapping feature spaces occupied by different phones represented by blocks of contiguous frames in phone classification tasks. We believe that this advantage might generalize to acoustical modeling for ASR.

An attractive approach to manifold learning is offered by the graph-embedding supervised dimensionality reduction framework, which is an extension of classical Linear Discriminant Analysis (LDA [16, 17]) (e.g. [18–20]). However, in all approaches to dimensionality reduction attention must be paid to the issue of finding a proper balance between the empirical risk (over-fitting of the high-dimensional data) and the structural risk (failing to meet the requirement that the resulting representations properly reflect the actual degrees of freedom in the physical process that generated the (speech) data). One way in which that balance can be controlled is by imposing additional constraints on the matrix that projects the high-dimensional raw representations into a lower-dimensional space. For example, if we assume that smooth trajectories in articulatory space should result in smooth trajectories of parameters in high-dimensional acoustic space, then one obvious additional requirement is that the weights assigned to corresponding elements of neighboring frames vary smoothly, in order to obtain smooth trajectories in low-dimensional space. From a computational point of view, the most obvious way for imposing additional requirements on the projection matrix is by means of introducing regularization terms, e.g. [13, 21].

In this paper we investigate the application of a series of regularization methods for increasing the smoothness of the projection matrices by their derivatives along the time axis, including the conventional Tikhonov regularizer [14, 21–24], first-order derivative [25], second-order derivative [23, 26] and fourth-order derivative which is also widely adopted as the penalty of smoothness [27].

Phone classification is a multi-class problem, with the number of classes C equal to the number of phone labels that are used in the labeling of a training corpus. Multi-class classification can be approached in three ways: by means
of a classifier that uses \( C \) models in parallel and assigns an unknown observation to the class \( c_i \) that returns the best match; by combining the results of \( C \) binary classifiers that separate class \( c_i, c = 1, 2, \ldots, C \) from all \( C-1 \) remaining classes, or by integrating the results of \( C(C-1) \) binary classifiers that separate all pairs of two classes. In this paper we focus on separating pairs of highly confusable classes [14, 21]. We opt for this form of binary classification because the results are useful for phonetic research and advanced acoustic modeling in ASR. Different pairs of classes, even within a broad phonetic class might need different features for their separation. Such a targeted approach is difficult or impossible to implement in a multi-class classification strategy.

The rest of this paper is organized as follows: In Section 2 we briefly introduce various regularizers, and explain how they impose smoothness constraints on the mapping into a lower-dimensional space by the specific variant of LDA that we use in this research, i.e., Locally Discriminant Embedding (LDE) [28]. In Section 3 we explain the design of the experiments. Section 4 presents the results of the experiments. Discussion and conclusions are presented in Section 5.

2. Regularization for Speech Trajectories

Our phone classification experiments are based on the TIMIT corpus [29], which comes with accurate labels and the matrix representation to a lower-dimensional space by the specific variant of LDA that maximizes the separation between the two classes. In the traditional LDA approach \( \mathbf{W} \) is found by

\[
\mathbf{W} = \arg\max_{\mathbf{W}} \left( \frac{\text{tr}(\mathbf{W}^T \mathbf{S}^{(b)} \mathbf{W})}{\text{tr}(\mathbf{W}^T \mathbf{S}^{(w)} \mathbf{W})} \right)
\]

In Eq. (4) \( \gamma (0 \leq \gamma \leq 1) \) is the weight given to the regularization term \( \mathbf{R} \). Different choices for the matrix \( \mathbf{R} \) result in different solutions for minimizing the structural risk.

When we represent phones as a block of 23 frames of 13 MFCC features we have a 299-dimensional space, in which we have no more than a couple of hundred tokens of each class. Thus, we have a small-sample-size problem [23, 24]. This means that the scatter matrices, especially the within-scatter matrix, are poorly estimated. This is likely to make the projection matrix \( \mathbf{W} \) difficult to interpret in physical terms. For instance, the inter-correlation of features along time, caused by the continuous movements of the articulators, is probably under-estimated. Moreover, some spurious structures, such as the correlation among MFCCs, which is expected to be re-appear in the scatter matrices. Ideally, to be interpretable, the projection matrix \( \mathbf{W} \), which can be considered as a set of basis vectors, should “match” the continuity properties of the input, in our case the smooth speech trajectories represented in \( \mathbf{X} \). Therefore, there is a necessity for regularizing the scatter matrix to be consistent with the underlying physical processes by smoothing the projection matrix along the time axis.

In the following we introduce four regularization terms to enhance the smoothness and therewith the interpretability of the projection matrix, by defining \( \mathbf{R} \) in four different ways.

2.2.1. Enhancing Interpretability by Tikhonov Regularization

The simplest way for imposing smoothness on the transformation matrix \( \mathbf{W} \) is by limiting its variance. This can be accomplished by using Ridge Regression Regularization [22] or Tikhonov Regularization [21, 23, 24] by setting \( \mathbf{R} = \mathbf{I} \), which means minimizing the \( L_2 \)-norm of \( \mathbf{W} \).

This regularizer effectively imposes an upper bound on the variance of the coefficients in the transformation matrix \( \mathbf{W} \), which is equivalent to biasing the estimator. When setting \( \mathbf{R} = \mathbf{I} \) and increasing the value of \( \gamma \), Eq. (4) will increase the bias in the estimate of the within-class scatter matrix. The bias will weaken the impact of eigenvectors with large eigenvalues, while the impact of eigenvectors with small eigenvalues will be strengthened. However, the Tikhonov regularizer can only decrease the overall variance; it cannot impose local smoothness.

For \( \gamma = 0 \), Eq. (4) defaults to the unbiased solution. The optimal value of \( \gamma \) for a specific application and data set must be found experimentally in a cross-validation design. This also holds for other choices of \( \mathbf{R} \).
2.2.2. Enhancing Continuity by Laplacian Smoothing

While the Tikhonov regularization reduces the overall variance in the transformation matrix, there is no guarantee that the derivatives of the sequence of coefficients along the time dimension are smooth. To realize this, the projection matrix $W$ must retain the time continuity of the trajectories. "Continuity" can be linked to the smoothness of the discriminant functions that make up $W$ by minimizing the derivatives of the row vectors (over time). The second-order derivative is the most commonly-used measure for the smoothness [23, 26, 27], which can be quantified in terms of the coefficients in row $m$ of the $M (= 13) \times N (= 23)$ matrix $W$:

$$J_2(w_m) = (w_{m1} - w_{m2})^2 + \sum_{i=2}^{N-2} (w_{mi} - 2w_{mi+1} + w_{mi+2})^2 \quad (5)$$

With this definition, the smoothness of the projection matrix $W$ is the sum of Eq. 5 over all rows: $J_2(W) = \sum_m J_2(w_m)$. This can be reformulated in matrix terms as $J_2(W) = W^T D_2 W$, where the $23 \times 23$ matrix $D_2$ is the so-called Laplacian smoother or Neumann discretizer [23, 26]:

$$D_2 = \frac{1}{h_2^2} \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ \vdots & \vdots & \vdots \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

The weight factor $1/h_2^2$ is related to the number of grid points on which the second order derivative along the time dimension is estimated. Substituting $D_2^{-1}$ for $R$ in Eq. 4 yields the so-called Smoothed LDE method using the 2nd-order derivatives (SLDE-2 method). Actually, the first-order and fourth-order derivatives are also widely used for smoothness penalty [25, 27]. They are obtained by replacing $D_2$ by $D_1$ for the first-order derivative and $D_4$ for the fourth-order derivative:

$$D_1 = \frac{1}{h_1^2} \begin{pmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ \vdots & \vdots & \vdots \\ -1 & 1 & 0 \end{pmatrix}$$

$$D_4 = \frac{1}{h_4^2} \begin{pmatrix} -1 & 1 & 0 \\ -2 & 5 & -4 \\ 1 & -4 & 6 \\ \vdots & \vdots & \vdots \\ 1 & -4 & 6 \\ 0 & 1 & -2 \end{pmatrix}$$

In fact, the Tikhonov regularizer that penalizes the norm of $W$ could be viewed as an approach to penalizing the zeroth-order derivative ($D_0 = I$). Therefore, in this paper, we will investigate four variants of the Smooth LDE (SLDE), named by the order of derivative it is based upon (SLDE-$n$, $n = 0, 1, 2, 4$). To be more specific, in SLDE-$n$ the matrix $R$ in Eq. (4) is replaced by $D_n D_n^T$.

3. Experimental Setup

3.1. The Data: TIMIT

In our phone classification experiments with TIMIT we adhered to the standard division of the corpus in training, testing, and tuning data [30]. We use the reduced label set proposed in [31]: the original 64 labels are collapsed into 48 phone labels, excluding the glottal stops. We reduce the number of relevant phone classes further, by excluding all forms of "silence"; this leaves us with 44 phone classes.

A short-time Fourier transform is performed on each utterance with a 25 millisecond Hamming window which is shifted in 10 millisecond steps. The Fourier coefficients are transformed into 13 MFCCs: $c_1 - c_{12}$. Phone tokens are represented by a block of 23 frames, whose center frame is aligned with that of the phone.

3.2. Classification Task

We investigate binary classification with $44 \times (44 - 1) / 2 = 946$ pairs of phone classes. Actually, a large number of classes are hardly ever confused. For example, the classification between vowels and voiceless consonants are likely to achieve (nearly) 100% accuracy. Even for some pairs from one broad phonetic class there are virtually no confusions. Therefore, we focused on the class pairs whose feature space overlap substantially; knowing how these pairs can be separated is crucial to understand the phonetic feature space. For this purpose, we define "confusability" as follows: for each phone pair, if any of those compared methods yields a classification accuracy lower than 0.80 or 0.95, that phone pair will be referred to as "highly confusible" or "fairly confusible", respectively.

3.3. Classification Strategy

In our experiments we first reduce the dimensionality of the phone representations, and subsequently use a weighted $k$ Nearest Neighbor (WkNN) classifier for the eventual binary classification task [10]. For any test vector $t$, we first find its $k$ nearest neighbors in the training set: $t_1, t_2, ..., t_k$. The weights of these neighbors are accumulated by $w_i = \exp(-||t - t_i||^2/\tau)$, $i = 1, 2, ..., k$, in which $\tau$ controls the influence of neighbors. The phone label assigned to $t$ is the class with the highest aggregated weights.

The LDE procedure for dimensionality reduction requires setting three parameters, viz., size of the between-class neighborhood $U_b$ and the within-class neighborhood $U_w$, as well as the number of dimensions $d$ of the target space. In [32] it was found that the classification performance is not sensitive to variations in the value of $U_b$ and $U_w$, as long as these parameters have values in a reasonable range. For the research in this paper we set $U_b = 20$ and $U_w = 6$. We also found that the optimal dimensionality of the target space is $d = 10$ for consonants and $d = 25$ for vowels. The results provided in the following section are based on these LDE settings. The $h$ in the prefactor of $D_4$ can be omitted: we normalize $S^2$ and $R$ to achieve an interpretable balance between them. The WkNN classifier has two parameters that must be set. In a number of preliminary
4. Experimental Results

4.1. Classification Per Broad Class

Since different broad phonetic classes probably contains phones produced by disparate mechanisms, this subsection reports the classification results for each broad phonetic class: Plosives (PL), Strong Fricatives (SF), Weak Fricatives (WF), Nasals (NS), Semi-Vowels (SeV), Short Vowels (ShV), and Long-Vowels (LoV). According to Subsection 3.2, only the “fairly confusable” pairs are concerned. Table 1 and Table 2 introduce the numbers of phone pairs and tokens for each broad phonetic classes. It should be mentioned that the number of tokens might be larger than the number of overall tokens in each broad phonetic class since parts of phones might be used more than once. For instance, the plosive /b/ is easily confused by /p/ and /g/, and thus the test samples of /b/ will be at least used twice.

Table 1: Statistics of the number of pairs and used tokens of the “fairly confusable” sets (≤ 0.95), for broad phonetic classes: Plosives (PL), Strong Fricatives (SF), Weak Fricatives (WF), Nasals (NS), Semi-Vowels (SeV), Short Vowels (ShV), and Long-Vowels (LoV)

<table>
<thead>
<tr>
<th>Nums</th>
<th>PL</th>
<th>SF</th>
<th>WF</th>
<th>NS</th>
<th>SeV</th>
<th>ShV</th>
<th>LoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pairs</td>
<td>11</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>Tokens</td>
<td>2841</td>
<td>941</td>
<td>857</td>
<td>1833</td>
<td>1373</td>
<td>6367</td>
<td>2649</td>
</tr>
</tbody>
</table>

Table 2: Statistics of the number of pairs and used tokens of the “highly confusable” sets (≤ 0.90), for broad phonetic classes: Plosives (PL), Strong Fricatives (SF), Weak Fricatives (WF), Nasals (NS), Semi-Vowels (SeV), Short Vowels (ShV), and Long-Vowels (LoV)

<table>
<thead>
<tr>
<th>Nums</th>
<th>PL</th>
<th>SF</th>
<th>WF</th>
<th>NS</th>
<th>SeV</th>
<th>ShV</th>
<th>LoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pairs</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Tokens</td>
<td>476</td>
<td>502</td>
<td>169</td>
<td>557</td>
<td>669</td>
<td>3149</td>
<td>634</td>
</tr>
</tbody>
</table>

Table 3: Performance comparison of the average binary classification accuracy of five methods: LDE and SLDE-\(n\) (\(n = 0, 1, 2, 4\)) on the fairly confusable pairs (≤ 0.95), for broad phonetic classes: Plosives (PL), Strong Fricatives (SF), Weak Fricatives (WF), Nasals (NS), Semi-Vowels (SeV), Short Vowels (ShV), and Long-Vowels (LoV)

<table>
<thead>
<tr>
<th></th>
<th>LDE</th>
<th>SLDE-0</th>
<th>SLDE-1</th>
<th>SLDE-2</th>
<th>SLDE-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PL</td>
<td>92.90</td>
<td>93.18</td>
<td>93.33</td>
<td>93.53</td>
<td>93.33</td>
</tr>
<tr>
<td>SF</td>
<td>91.32</td>
<td>90.92</td>
<td>91.59</td>
<td>91.45</td>
<td>91.60</td>
</tr>
<tr>
<td>WF</td>
<td>91.72</td>
<td>92.51</td>
<td>93.26</td>
<td>93.28</td>
<td>93.10</td>
</tr>
<tr>
<td>NS</td>
<td>89.69</td>
<td>90.18</td>
<td>90.27</td>
<td>90.37</td>
<td>90.63</td>
</tr>
<tr>
<td>SeV</td>
<td>91.59</td>
<td>92.35</td>
<td>92.72</td>
<td>93.10</td>
<td>92.99</td>
</tr>
<tr>
<td>ShV</td>
<td>89.29</td>
<td>89.70</td>
<td>89.87</td>
<td>89.73</td>
<td>89.85</td>
</tr>
<tr>
<td>LoV</td>
<td>92.46</td>
<td>93.37</td>
<td>93.66</td>
<td>93.34</td>
<td>93.53</td>
</tr>
</tbody>
</table>

Table 4: Performance comparison of the average binary classification accuracy of five methods: LDE and SLDE-\(n\) (\(n = 0, 1, 2, 4\)) on the highly confusable pairs (≤ 0.90), for broad phonetic classes: Plosives (PL), Strong Fricatives (SF), Weak Fricatives (WF), Nasals (NS), Semi-Vowels (SeV), Short Vowels (ShV), and Long-Vowels (LoV)

<table>
<thead>
<tr>
<th></th>
<th>LDE</th>
<th>SLDE-0</th>
<th>SLDE-1</th>
<th>SLDE-2</th>
<th>SLDE-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PL</td>
<td>88.13</td>
<td>88.76</td>
<td>89.02</td>
<td>89.56</td>
<td>89.36</td>
</tr>
<tr>
<td>SF</td>
<td>88.05</td>
<td>87.05</td>
<td>88.25</td>
<td>88.25</td>
<td>88.25</td>
</tr>
<tr>
<td>WF</td>
<td>85.80</td>
<td>88.76</td>
<td>89.92</td>
<td>89.15</td>
<td>89.66</td>
</tr>
<tr>
<td>NS</td>
<td>81.51</td>
<td>82.94</td>
<td>83.30</td>
<td>83.30</td>
<td>83.75</td>
</tr>
<tr>
<td>SeV</td>
<td>88.34</td>
<td>89.12</td>
<td>89.54</td>
<td>90.47</td>
<td>90.06</td>
</tr>
<tr>
<td>ShV</td>
<td>85.28</td>
<td>85.75</td>
<td>85.88</td>
<td>85.60</td>
<td>85.75</td>
</tr>
<tr>
<td>LoV</td>
<td>86.04</td>
<td>86.91</td>
<td>86.69</td>
<td>86.67</td>
<td>86.91</td>
</tr>
</tbody>
</table>

4.2. Analysis on the Tuning Parameter \(\gamma\)

An alternative to analyze the effectiveness of regularizers is to explore how the performance varies as a function of the weighing parameter \(\gamma\). This is shown in Fig. 1 for consonants and Fig. 2 for vowels.
Both figures refine our earlier observation that adding the regularization term is beneficial to LDE (LDE corresponds to $\gamma = 0$). Moreover, the performance of “SLDE-0” is comparable with that of other three regularization terms for vowels but not for consonants. This can be deduced from the observation that the peak region of “SLDE-0” (the blue curve in Fig. 2) stays on a similar horizontal level as the other curves do. However, for consonants, the peaks of “SLDE-n” ($n \geq 0$) are consistently higher than those of “SLDE-0” (c.f., Fig. 1). The figures show that the three methods “SLDE-n” ($n > 0$) achieve similar performance.

The three smoothness regularizers have different meaningful $\gamma$-ranges, where their performance is superior to the original LDE ($\gamma = 0$). Since all these regularizers adopt the same exhaustive tuning grid and all related matrices ($S^w$ and different $R$s) are normalized, we argue that the higher the order of the derivative of $W$ we use, the most robust the performance is. More precisely, SLDE-4 (corresponding to the magenta curves with “diamonds”) outperforms LDE for almost all values of $\gamma$, while for $n < 4$ “SLDE-n” has a narrower optimal $\gamma$-range.

In summary, this sensitivity analysis further substantiates the effectiveness of introducing $R$ into LDE and indicates the robustness for variations in $\gamma$ when applying the fourth-order derivative of $W$ for regularization.

### 4.3. The Impact of $R$ on $(S^w + \gamma R)^{-1}$

In this part, the deeper analysis on the impact of the regularization term is given by showing the structural alteration of the inverse matrix of the regularized within-class scatter matrix: $(S^w + \gamma R)^{-1}$. This is due to the fact that $(S^w + \gamma R)^{-1}S^w$ directly determines the projection matrix $W$. Fig. 3 is an example with $R = D_2D_2^T$. We choose $D_2$ to guarantee the inferior results when $\gamma$ approaches 0 and 1. Four images in this figure are $23 \times 23$, which correspond to the counterpart of $(S^w + \gamma R)^{-1}$ of the first MFCCs of 23 frames. As indicated in the figure, the upper-left one is the original LDE, meaning $\gamma = 0$. The upper-right and the lower-left images are with the small and medium $\gamma$s. When $\gamma$ increases (from “zero” to “small” to “medium”), the corresponding image will have a wider “band”, which means the enhancement of the structures over time. Referring to Fig. 1 and Fig. 2, the time-trajectory structure might explain the gain achieved by enlarging $\gamma$.

However, the lower-right image, whose $\gamma$ is large enough to suppress the contribution of $S^w$ when computing $(S^w + \gamma R)^{-1}$, almost loses the structure of $(S^w)^{-1}$, especially on its diagonal part. This might explain the (great) inferior performance when $\gamma$ approaches 1 in Fig. 1 and Fig. 2.

Therefore, it might be concluded the impact of $R$ on $S^w$ is simultaneously enhancing the time-trajectory structure and retaining the details of $S^w$ with the trade-off parameter $\gamma$.

### 5. General Discussion and Conclusion

To model the time-continuity of speech trajectories for the purpose of phonetic classification, this paper introduces the idea of smoothing the projection matrix of Linear Discriminant Embedding [28], a supervised dimensionality reduction method adopting the manifold information which was proven superior over the conventional Linear Discriminant Analysis. The basic idea is realized by penalizing $W$’s derivatives to enhance the smoothness of the underlying manifold. Specifically, four orders of derivatives are investigated in this paper, namely the zeroth-order (Tikhonov regularizer), first-order, second-order, and fourth-order derivatives. All of them are implemented by adding the specified regularization term $R$ to the within-class
scattering matrix $S^w$. The binary classification performance on the confusable phone pairs briefly reveals the effectiveness of imposing the regularization terms. When comparing the four alternatives, the zeroth-order one appears to be moderately inferior to the other three comparable regularizers for consonants but yield competitive classification accuracy in the case of vowels. This might indicate a larger necessity of exploiting the continuity within the trajectories for consonants compared to vowels. The robustness of $\gamma$ in Eq. (4) is also explored. Both Fig. 1 and Fig. 2 indicate that higher-order derivatives can generate a more robust regularizer: using the fourth-order derivative results in improvement over LDE for nearly all $\gamma$s in the $[0, 1]$-grid. More importantly, the reason why the regularization term $R$ leads to improvement is likely to be related to how $R$ influences structures of $(S^w)^{-1}$ over time (i.e., “frame number” in Fig. 3). The observation is that tuning $\gamma$ should enhance the time-trajectory structure without losing too much the details in $S^w$. The analysis on $(S^w + \gamma R)^{-1}$ suggests that regularization is an effective way to integrate prior information into a small-sample-size problem.

In the near future, our research aim is to analyze the speech production mechanisms, relate them with the smoothness regularizer and thus get more insight into the feature space of speech trajectories. Another interesting topic is to integrate this approach with ASR. Probably regularization is useful for noise-robust ASR since the time-continuity of speech might be robust to environmental noise. Finally, a theoretical analysis on how exactly $R$ perturbs the eigenvectors of $(S^w + \gamma R)^{-1}S^w$ will be a research topic in the near future.

6. References